APPENDIX B

Formative Assessment for PK-3 Mathematics

A Review of the Literature

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Purpose of the review

The development and implementation of formative assessment systems are hampered by a lack of clarity regarding what formative assessment is and what purposes it can and cannot serve (Barton, 2004) and by a lack of rigorous, empirical evidence to support claims that it has a positive impact on student achievement (Dunn & Mulvenon, 2009). The purpose of this literature review is to inform the development of a formative assessment system for mathematics to be used with Florida students in grades PK-3. The review addresses the following questions:

(1) What is formative assessment?
(2) What formative assessment characteristics and practices are recommended in the literature?
(3) How do learning progressions and feedback affect the quality of formative assessments?
(4) What formative assessment tasks are used for mathematics in PK-3 classrooms?
(5) What foundational knowledge and understanding are most predictive of PK-3 students' future mathematics learning and achievement?

What is formative assessment?

Perie, Marion, and Gong (2007) believe that even assessment experts suffer from the lack of definitions for different types of assessments. Most assessments, even summative assessments, can be used for formative purposes, such as diagnosis, prediction, and evaluating teacher and/or student performance. In an attempt to alleviate some of the confusion surrounding formative assessment, the Council of Chief State School Officers (CCSSO) formed the Formative Assessment Advisory Group in 2006. The Advisory Group included experts in measurement and educational research as well as state agency leaders from across the country (CCSSO, 2008). This advisory group was charged with clarifying the definition of *formative assessment* and determining how educators could use formative assessment most effectively. The following is the Group's definition:

> Formative assessment is a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes. (p. 3)

This definition of formative assessment is used throughout this literature review. Moreover, our discussion will be limited to assessments matching the CCSSO's definition and will, therefore, exclude the following types of assessments:

- Screening: assessments designed to identify students likely to experience success or difficulty in learning with well-planned instruction
- Broad diagnostic: assessments designed to identify key areas of potential difficulty in learning. For example, while screening may focus on numeration, the broad diagnostic measure might be spatial reasoning.
- Targeted diagnostic: assessments for students identified as at risk for learning difficulties. Such assessments isolate areas of students' misconceptions.
- Progress monitoring: regularly administered assessments of students' knowledge to evaluate the effectiveness of instruction.

As the CCSSO definition states, formative assessment is a process not a test. A formative assessment "is embedded within the learning activity and linked directly to the current unit of instruction" (Perie et al., 2007, p. 13). Often classroom assessments are administered outside of routine learning activities, are scored and analyzed outside of the classroom, and *inform* later instructional activities. Formative assessment is "in-the-moment assessment" (Marshall, 2008, p. 68). Teachers receive evidence of student knowledge and understanding and provide immediate feedback to students and adapt the learning tasks to be more aligned with the critical needs identified in the assessment. Instead of providing a snapshot of learning, formative assessments create opportunities to advance learning; they are one of the components of a comprehensive unit of study that targets specific gaps and weaknesses in knowledge and misconceptions that undermine students' learning.

For all the benefits of formative assessments, they do have limits that need to be considered in choosing amongst the variety of classroom assessment formats. Most classroom assessments are designed to capture student performance data that can be used to enhance instruction. Formative assessments, however, are not easily aggregated beyond the classroom level or reported at a broader level, such as district or state level. They provide evidence of immediate learning but do not necessarily measure learning retention weeks or months after initial instruction occurs. While relevant for instructional purposes, formative assessments are not as relevant for evaluative or predictive purposes. For example, they are not necessarily predictive of students' performance on summative or end-of-unit tests (Marshall, 2008). Furthermore, establishing validity is more complicated because of the instructional component (Stobart, 2008). Formative assessments are "often implemented in a non-standardized and hence less rigorous manner than summative assessment, and thus can hamper the validity and reliability of the assessment tools and data" (Shute & Zapata-Rivera, in press).

The feedback that instructors provide following classroom assessment is a critical component of formative assessment. Outcomes regarding student learning are directly influenced by this feedback, so it is difficult to separate the assessment's influence from the feedback/instruction's influence on student learning. "It is difficult to parse out formative assessment effects from teacher effects" (Dunn & Mulvenon, 2009). It is also difficult to control the content of the feedback and the manner in which teachers provide feedback to students. The type, timing, and degree of specificity and/or complexity of feedback have the potential to impact the feedback's affect on student learning. The content and administration of the assessment may be controlled and fidelity of implementation monitored, but it is more difficult to ensure the fidelity and replication of individual instructor's feedback and post-assessment instruction.

Determining whether formative assessment fosters student learning is complicated by this union of assessment and instruction that is the hallmark of the model. In their recent review of research, Dunn and Mulvenon (2009) concluded that the research does not provide conclusive, empirical evidence that formative assessment positively affects student learning and achievement. They call into question Black & Wiliam's (1998) seminal work and its claim that classroom assessments for learning, such as formative assessment, have a positive impact on student achievement. Black & Wiliam's (1998) reviewed some 250 articles by researchers from several countries (Barton, 2004). Dunn and Mulvenon (2009) suggest that methodological weaknesses and limitations in the eight studies Black and Wiliam (1998) present to evidence their claim undermine their positive finding. One of the weaknesses cited was the high percentage of students studied who were from the special education population. In Fuch and Fuch's (1986) meta-analysis, for example, 83 percent of the 3,835 participating students were "handicapped" according to Dunn & Mulvenon (2009, p. 5). Other concerns were the lack of generalizability and the overall poor quality of the research (Dunn & Mulvenon, 2009). Dunn and Mulvenon (2009) also reviewed nine more recent research articles and concluded that the support for formative assessment's effectiveness remains questionable because of methodological weaknesses similar to those they identified in the Black & Wiliam (1998) article. However, the degree to which these methodological short-comings affect the authors' findings is not clear.

Rigorous, empirical research is needed to determine conclusively whether or not formative assessment positively influences student learning and if so what key attributes are responsible for its success. "A sound research-validated framework for best practices in formative assessment" is crucial to maximizing the potential benefits of this assessment and instructional model (Dunn & Mulvenon, 2009, p. 9). Are improvements in student learning related to the quality of the assessment or the quality of the subsequent instruction? Despite the methodological challenges and weaknesses, the literature does demonstrate the potential for formative assessments (as an integrative system of assessment) to improve student learning and offers guidance regarding optimal strategies and practices to maximize that potential.

What formative assessment characteristics and practices are recommended in the literature?

Wiliam and Thompson (2007) suggest that a comprehensive framework for formative assessment should include three central processes: (a) "establishing where students are in their learning, (b) establishing where they are going, and (c) establishing how to get there" (p.1). The following is a compilation of six key strategies and attributes of effective formative assessment identified by the National Council of Teachers of Mathematics (NCTM) and CCSSO.

Identify the learning goals and criteria for success and clearly explain these to learners

Wiggins and McTighe (2000) advise teachers first to identify learning goals—"what is worthy and requiring understanding"—and then to establish the criteria for success. Only after these have been determined should the teacher design the activities to assess students' current knowledge and understanding of the goal and to provide opportunities to advance their progress toward mastery of the goal. The instructional goals and criteria the teacher will use to assess student success need to be clearly communicated to students. Students need to understand the criteria thoroughly so that they can assess their own progress toward achieving those criteria. Discrepancies in students' "beliefs about what it is that counts as learning in mathematics classrooms may be a significant factor in the achievement gaps observed in mathematics classrooms" (Wiliam, 2007, p. 1). To enhance student understanding of the criteria, the teacher might provide students with examples of a successful, high-quality product (e.g., essay, drawing, analysis of an argument). These examples enable students to learn from expert/novice modeling.

Construct learning progressions from the learning goals

Constructing learning progressions involves breaking the overall learning goal into short-term subgoals. The construction of the progressions will be discussed in detail later, but the benefit of such progressions is that the core knowledge and understanding students must master are presented as a continuum of development; progressions reflect the understanding that students learn at different rates and straddle grade-level expectations. As learning progressions are constructed by different people, there will be variations in the content and sequencing of skills and concepts (CCSSO, 2008a).

Design effective classroom discussions, questions, activities, and tasks that elicit evidence of students' learning

To elicit the evidence needed to revise and refine instruction, teachers need to plan the right kinds of activities, tasks, questions, etc. to reveal students' knowledge and understanding of the learning that is the goal.

Provide descriptive feedback that moves learning forward

Research indicates that the feedback given to students often has little impact and can even be detrimental to learning (Shute, 2008). Teachers should provide students evidence-based feedback that is directly linked to the learning outcome and criteria for success. Effective feedback needs to focus on the quality of student learning rather than on the student or the accuracy of the student's response. The feedback should also provide suggestions for how the student can improve. Feedback "should help the student answer three basic questions: Where am I going? Where am I now? How can I close the gap?" (CCSSO, 2008a, p. 4). Feedback should also force students to engage cognitively; finding errors themselves, reflecting on teacher and peer feedback, and acting on that feedback.

Encourage self- and peer-assessment

A goal of formative assessment is to encourage and model meta-cognitive thinking so that students understand they are responsible for their own learning and actively participate in evaluating and monitoring their progress in relation to the criteria. Research shows that when students take an active role in "monitoring and regulating their learning, then the rate of their learning is dramatically increased," in fact in some instances doubled (Wiliam, 2007, p. 3). Students need to reflect on the feedback they receive from their teachers or peers. Slavin, Hurley, and Chamberlain (2003) found that when students serve as learning resources for each other large gains in learning occur. For these gains to take place, however, two elements are critical: students must work *as* a group not just *in* a group, and each student must be held accountable for his or her contribution to the group. Students can use guidelines and/or rubrics to help them provide feedback that is directly linked to the criteria and that can foster improvement. Evaluating peer work and providing productive feedback allows students to practice and build skill in improving their own work.

Establish a classroom environment that facilitates collaboration

A nonthreatening classroom environment is essential for formative assessments to enhance learning. In an optimal classroom, students trust their teacher as well as their peers and differences in ideas and opinions are respected and appreciated. To create such an environment, teachers need to model such behaviors and demonstrate how to provide appropriate feedback (Wiliam, 2007; CCSSO, 2008a).

With the guidance of Caroline Wylie at ETS, the CCSSO's State Collaborative on Assessment and Student Standards developed a companion document to assist teachers in implementing its five characteristics of effective formative assessment: learning progressions, learning goals and criteria for success, descriptive feedback, self- and peer-assessment, and collaboration (CCSSO, 2008b). *Formative Assessment: Examples of Practice* (CCSSO, 2008b) begins with vignettes of classroom practices and asks the reader whether the practice is or is not an example of formative assessment. Examples emphasize the following key attributes of formative assessments:

- Teachers embed the assessment in a learning activity.
- Teachers collect evidence of student understanding by observing students performance during the learning activity and evaluating the performance against the established criteria for success.
- Teachers and/or other students provide feedback directly related to the criteria.
- Teachers encourage student metacognition and reflection on feedback.

A critical component of quality formative assessment is teachers' use of the evidence obtained from students' performance on assessment tasks to *adjust instruction* and to *guide students in adjusting their learning strategies*. To determine how to adjust instruction, teachers interpret the assessment evidence to pinpoint where along a learning progression or trajectory students' learning stopped progressing. Teachers may then select from a variety of feedback types and strategies to provide the instruction needed to get the student's learning back on track and moving forward toward accomplishing the learning goal.

How do learning progressions and feedback affect the quality of formative assessments?

The literature describing the key components of formative assessment consistently emphasizes the need for clear learning goals, detailed learning progressions related to these goals, and feedback that advances students. Constructing learning progressions involves drilling below the skill or concept level to identify the foundational cognitive substructures and sub-concepts required for meaningful and sustainable skill and concept development. These progressions enable teachers to more precisely and directly target feedback at gaps and weaknesses in these foundational competencies.

Learning progressions

The literature on learning progressions describes learning as “a development of progressive sophistication in understanding and skills within a domain” (Heritage, 2008, p. 4). Learning progressions for a domain connect the knowledge, concepts, and skills students develop as they evolve from “novice to more expert performance” (p. 4). According to Heritage (2008), learning progressions are crucial to the three principal elements of formative assessment:

- Eliciting evidence about learning to close the gap between current and desired performance
- Providing feedback to students
- Involving students in the assessment and learning process.

Both the NCTM and CCSSO descriptions of quality formative assessments begin with the need for clear and specific learning goals and criteria for determining the student’s level of success in accomplishing those goals. Learning goals can be broken down or decomposed into core sub-concepts students need to learn in order to progress towards the broader learning goal. Establishing a progression of sub-concepts that scaffold continued learning enables teachers to see the larger picture of what they want their students to learn and the conceptual and procedural mile markers that indicate movement towards that broader goal. Formative assessment should be directed at those sub-concepts or mile markers so that the assessment and resulting instruction will be sufficiently precise, specific, and directed. Learning involves cognitive construction. To build a solid and stable structure, the foundation as well as each additional component needs to be solid and stable. A weak or unbalanced cognitive structure will not adequately support additional and more complex learning. By assessing the strength of each sub-concept, teachers are better able to target instruction precisely to ensure that student’s foundational concepts are adequate and resilient enough to support the next sub-concept in the learning progression. In addition to learning critical sub-concepts, students can “come to see and understand the scaffolding they will be climbing as they approach” their learning goals (Stiggins, 2005, p. 327).

The development and use of learning progressions is “an extension of the standards-assessment model for educational reform.” Standards, however, “typically lack the detail, developmental coherence, and skills (reasoning, etc.) needed to present sufficient learning targets and learning progressions” (Gong, 2007). Some standards do not clearly describe the “core concepts and sub-concepts that scaffold student’s growth from one standard to another”; consequently, “teachers are not able to determine where student learning lies on a continuum, and know what to do to close the gap between current learning and desired goals” (Heritage, 2008, p. 2). To make comprehensive instructional decisions, teachers need to understand “the core building blocks of learning,” how they are connected and sequenced, and how they “develop from the earliest to the most sophisticated level” (Heritage, 2008, p. 2; Honey, 2007). By identifying where student knowledge and understanding lie on this progression of learning, teachers are better able to design instruction that directly targets misconceptions and weaknesses that prevent advancement toward the overall learning goal and create or strengthen the understanding and skills, the footholds, needed to climb to the next level of performance.

Learning progressions may align with grade-level benchmarks, but the progressions are not necessarily segmented by grade level. If a student has not mastered a concept—regardless of the grade-level at which the concept was introduced—the teacher can look for the point along the progression where the student’s understanding failed and/or misunderstanding began and new learning stopped. Instruction can then be directed at this point in the sequence where learning derailed instead of repeating instruction on the larger concept, which the student may not even possess the cognitive or knowledge foundation to understand. Even if the student can eventually perform an operation and achieve competency on a test, the gap in conceptual understanding acts as a crack in the foundation of learning and weakens the overall structure. At some point, the structure will collapse and future learning will be jeopardized.

Two examples of learning progressions are included in Appendix C. One is a *Counting and Ordering Progress Map*, and the other—*A Developmental Model for Learning Functions*— is a learning progression that can also function as a unit of study. As part of its Project 2061, a professional development of educator’s project, the

American Association for the Advancement of Science (AAAS) created an Atlas of Science Literacy. The Atlas presents approximately 100 strand maps that graphically display how students progress in their understanding and show the conceptual connections between the ideas and skills that students need to learn as they progress toward literacy (AAAS, 2009). These strand maps serve as models for constructing learning progressions (Gong, 2007).

Heritage (2008) suggests that "the development of user-friendly models of student progression in learning, where clear targets for instruction and assessments are identified" is necessary to attain the National Research Council's (NRC) vision of a system of assessment—one that has coherence, comprehensiveness, and continuity (p. 12). "Current research only defines how a limited number of areas can be divided into learning progressions." Heritage offers two general strategies for constructing learning progressions: *top-down* and *bottom-up*.

> In a top-down progression, experts in the domain (e.g., mathematicians) and other experts, such as developmental specialists, construct a progression based on their domain and research knowledge. The resulting progression represents their decisions about what constitutes the 'big ideas' of the domain and how they connect together. A bottom-up approach involves curriculum content experts and teachers in developing a progression that is based on their experience of teaching children. Their sources for developing the progression are curricula, their views of what is best taught when, and their knowledge of children's learning. (Heritage, 2008, p. 12)

The following is an example of a bottom-up approach that the Wisconsin Department of Education used in developing a learning progression for reading:

1. Teams of "curriculum content experts who had a district-wide and school-wide role and current classroom teachers (elementary, middle, and high school)" were established. These teams reviewed the state content standards and identified the subcomponents (Heritage, 2008, p. 13).
2. The teams worked collaboratively "to identify sub skills and sub concepts that would lead to understanding of the concept or acquisition of the skill" (Heritage, 2008, p. 13).

3. The teams then laid the sub-skills and sub-concepts "out in a progression that was logical for them and made sense in terms of what they knew about learning and instruction" (Heritage, 2008, p. 13).

One of the difficulties encountered during this process was that teachers—no matter their level of expertise or experience—consistently confused learning goals with how learning would be achieved. "For example, teachers identified creating area models as the learning goal rather than viewing them as a means for developing student understanding of the concept of equivalent fractions." As the teachers continued to work on the progression and to think more deeply about student learning, they were better able to distinguish the two. Another challenge was deciding on the level of detail required to enable teachers to use these progressions in their instruction and formative assessment. The Wisconsin group decided they were unable to determine precisely the level of detail needed and would adjust the progressions as they were used.

Assessment tasks included in a formative assessment should reflect a comprehensive learning progression to ensure that the student performance data teachers obtain show the degree of student understanding of core building blocks for future learning. Teachers can use the progression to inform or form instructional feedback that targets the most essential competencies.

Feedback

Quality feedback is a critical component of effective formative assessment. As these assessments are embedded in instructional activities and tasks, feedback provides the opportunity to introduce instruction in response to the evidence collected through student responses to the assessment. In a recent article, Shute (2008) provides a detailed and thorough review of the research on formative feedback—the type of feedback used in formative assessments as well as other *assessments for learning*. Her guidelines for the construction and implementation of formative feedback are included in Appendix F.

Formative feedback is "information communicated to the learner that is intended to modify his or her thinking or behavior for the purpose of improving learning" (Shute, 2008, p. 154). Research indicates that good feedback has the capacity to improve learning significantly, but success depends on how effectively the feedback is delivered (Shute, 2008). As formative assessment focuses on task-level assessment, good task-level feedback is particularly important. Task-level feedback

> provides more specific and timely (often real time) information to the student about a particular response to a problem or task compared to summary feedback and may additionally take into account the student's current understanding and ability level. (Shute, 2008, p. 154)

Formative feedback can provide learners the information they need to advance their own achievement. For example, feedback indicates the gap between the learner's current performance level compared to a specific performance goal. Such information reduces the student's uncertainty about his or her performance and progress (Ashford, 1986; Ashford, Blatt, & VandeWalle, 2003). Reducing the student's uncertainty can lead to greater motivation and more efficient strategies for solving problems and performing tasks (Kanfer & Ackerman, 1989). Learners, especially struggling learners who can become overwhelmed with the cognitive complexity of a task, may benefit from supportive formative feedback because it can reduce the learner's cognitive load. Feedback can also provide information that can assist learners in "correcting inappropriate task strategies, procedural errors, and misconceptions" (Shute, 2008, p. 157).

The following characteristics of formative feedback can affect its impact on student learning: (a) the type of information provided (verification or elaboration); (b) the complexity of the information; (c) the amount or length of the information; (d) the relation to specified performance goals; and (e) the timing of the feedback (immediate or delayed) (Shute, 2008).

Verification tells students whether or not their answers are correct. Elaboration provides information to guide students towards the correct answer. Research indicates that quality feedback should include elements of both types of information. One type of elaboration, response-specific feedback, appears to enhance student achievement, particularly learning efficiency, more than other types of feedback, such as simple verification or 'answer until correct'" (Shute, 2008, p. 159). Research results regarding the optimal degree of specificity or length of feedback are somewhat conflicting and therefore inconclusive. Although greater feedback specificity seems to be better than less specificity, more complex and/or longer feedback either has no effect or can have a negative effect on learning (Shute, 2008).

Goal-directed feedback provides learners information about their progress towards attaining a specific goal or set of goals rather than their performance on discrete assessment items or tasks. Goal-directed feedback is a natural outcome and component of formative assessment. The attributes of quality formative assessment include established learning goals and clear explanations of the goals and criteria for students to meet those goals. Feedback referencing the criteria and refocusing students on the goal enhances the assessments potential to improve learning and performance.

For students to remain motivated and engaged, literature suggests that performance goals need to be personally meaningful to students, and there needs to be an expectation that they can be attained. Bransford, Brown, and Cocking (2000) suggest that "a goal-directed approach to learning using scaffolding (or scaffolded feedback)

- Motivates the learner's interest related to the task
- Simplifies the task to make it more manageable and achievable
- Provides some direction to help the learner focus on achieving the goal
- Clearly indicates the differences between the learner's work and the standard or desired solution
- Reduces frustration and risk
- Models and clearly defines the expectations (goals) of the activity to be performed" (Shute, 2008, p. 163).

In regards to motivation, norm-referenced feedback that compares the learner's performance to other's performance can diminish poor performers' motivation. They tend to attribute their failure to their lack of ability and expect to perform poorly in the future. This finding suggests that low-achieving students should not be given normative feedback but rather feedback that focuses on their personal, individual progress (Shute, 2008).

How the timing of feedback influences learning is particularly relevant to formative assessment. Formative feedback—like formative assessment—is embedded in instructional activities and is generally provided immediately after a student responds to a question or performs a task. Research provides no conclusive evidence whether immediate or delayed feedback is more effective, but opinions seem to hinge on the dynamics of memory, the types of content to be learned—concept formation or procedural skills—and whether the learning goal is acquisition or transfer of knowledge. "Some researchers have argued for immediate feedback as a means to prevent errors being encoded into memory, whereas others have argued that delayed feedback reduces proactive inference, thus allowing the initial error to be forgotten and the correct information to be encoded with no interference" (Shute, 2008, pp. 163-164). Research findings also suggest that "delayed feedback may be superior for promoting transfer of learning, especially in relation to concept-formation tasks, whereas immediate feedback may be more efficient, particularly in the short-run and for procedural skills" (p. 165). Mason and Bruning (2001) recommend that immediate feedback is superior to delayed feedback for low-achieving students "in the context of either simple (lower level) or complex (higher level) tasks," while delayed feedback is optimal for high achieving students, especially for complex tasks (Shute, 2008).

Shute (2008) summarized four influential articles aimed at forming disparate findings from research into models or frameworks for creating effective assessments. The authors of the articles are Kluger and DeNisis (1996), Bangert-Drowns, Kulik, Kulik, and Morgan (1991), Narciss and Huth (2004), and Mason and Bruning (2001). Key suggestions from these studies include the following.

Bangert-Drowns et al. (1991) claim that feedback has the potential to promote learning if the learner receives it mindfully (i.e., reflects on the feedback and "explores situational cues and underlying meanings relevant to the task involved" (Dempsey et al., 1193, p. 38)). Narciss and Huth (2004) suggest that effective formative feedback must take into consideration instructional content for complex learning tasks. To create effective formative feedback, concrete learning outcomes and learning tasks matching those outcomes need to be identified as well as specific errors and difficulties (Shute 2008).

Kluger and DeNisi's (1996) meta-analysis found that "in more than one third of 607 cases, feedback interventions reduced performance" (Shute, 2008, p. 170). The authors' formative intervention (FI) theory recommends feedback that focuses the learner on aspects of the task as opposed to the learner's self. They argue that changing the locus of the learner's attention can affect performance and behavior, and that a learner's attention is limited and should be focused on the differences in the learner's performance and the goal or desired level of performance. Findings of the authors' meta-analysis suggest that the following decrease feedback's positive effect:

- Feedback that is discouraging or praises the student
- Feedback that threatens self-esteem
- Orally delivered feedback from the instructor
- Feedback in the context of complex tasks as opposed to simpler tasks
- An absence of goal-setting

Characteristics of feedback that increase positive effect include:

- Providing the correct response to the student
- Providing frequent messages
- Using computerized feedback delivery

Furthermore, feedback effects are stronger for cognitive tasks rather than physical tasks and for memory tasks rather than for procedural tasks.

Shute, Hansen, and Almond (2008) conducted a study to examine whether including feedback in an assessment system "(a) impairs the quality of the assessment (relative to validity, reliability, and efficiency), and (b) does, in fact, enhance student learning" (Shute et al., 2008, p 289). The assessment system for learning (a formative assessment model) they designed is called ACED (Adaptive Content with Evidence-based Diagnosis), and it includes an adaptive, computer-based diagnostic assessment and elaborated feedback to provide instructional support to learners. When designing their study, they concluded that task-level feedback—feedback given "right after a student has finished solving a problem or task"—and adaptive sequencing of tasks were most effective (Shute et al., 2008, p 291). Adaptive sequencing, as opposed to linear or fixed sequencing of tasks or items, adjusts the sequencing of tasks based on considerations such as which task would be best for estimating a student's level of competency and which task would be most effective at supporting the student's advancement to a higher level of competency (Shute et al., 2008).

The researchers examined varying conditions: (a) elaborated feedback with adaptive sequencing, (b) simple feedback with adaptive sequencing, and (c) elaborated feedback and linear sequencing. The elaborated feedback with adaptive sequencing was most effective. The control group of students completed pre- and post-tests but did not receive the intervention. They were instructed to sit at their desks and read material unrelated to mathematics while the intervention group completed the intervention assessment on computers (Shute et al., 2008). The results from the study "showed that the quality of the assessment was unimpaired by the provision of feedback. Moreover, the students using the ACED system showed significantly greater learning of the content compared with a control group" (p. 289). ACED proved to be successful as "both a valid and reliable assessment and a learning aid" (p. 309). The ACED model represents the dual components of a formative assessment, and the results of the study indicate that classroom, task-level assessments can be designed to provide quality evidence of student learning while fostering learning through the use of well-designed assessment tasks and instructional feedback.

What formative assessment tasks are used for mathematics in PK-3 classrooms?

Schoenfeld (2007b) describes the mathematical knowledge and understanding that characterize mathematical proficiency and provides examples of questions, tasks, probes, and scaffolding to assess and foster students' development towards mathematical proficiency. Although most of the assessment examples are for students beyond third grade, the competencies and the strategies for structuring assessments are relevant to assessment of younger children. The assessment strategies, which indicate how to access and reveal student thinking, are relevant to the assessment of younger children as well. The examples also illustrate strategies for extending items and scaffolding items to address higher levels of complexity.

Burkhardt (2007) believes that a well-designed assessment of mathematical proficiency measures what is important—rather than just what is easy to assess—and measures the quality of the students' overall performance not just accuracy at executing individual components. "Whenever curriculum and assessment choices are to be made, discussion should focus on performance as a whole, not just the range of mathematical topics to be included" (Burkhardt, 2007, p. 89). The Mathematics Resource Service created a *Framework for Balance* presenting four dimensions of performance that need to be balanced in an assessment: mathematical content, mathematical process, task type, and circumstances of performance. (The *Framework for Balance* can be found in Appendix D.) Designers seek to balance content throughout an assessment, but often only one type of performance task is included and these tasks are typically "short exercises that require only transforming and manipulating…. The ability to formulate a problem is trivialized, and interpretation, critical evaluation and communication of results and reasoning are rarely assessed" (Burkhardt, 2007, p. 89). To improve an assessment of mathematics, Burkhardt (2007) suggests beginning with the tasks. "Get excellent task designers to design and develop a wide range of good mathematics tasks, classify them with a domain model, and fill any major gaps needed to balance each test"(p. 95).

Burkhardt (2007) provides examples of assessment tasks: open investigation (students have to formulate questions as well as answer them), design tasks, planning tasks, nonroutine problems often based on pattern generalization, evaluate and recommend tasks, and "re-presentation" of information tasks. Most of the tasks were not designed for use with young children, but Burkhardt did include a re-presentation of information task designed to assess geometry and mathematical communication for third-graders. See Appendix E for a description of the task and the scoring rubric.

De Lange (2007) suggests the following types of assessment tasks for increasingly complex levels of mathematics competency:

- "Reproduction: simple or routine computations, definitions, and (one-step or familiar) problems that need almost no mathematization.
- Connections: somewhat more complex problem solving that involves making connections (between different mathematical domains and between the mathematics and the context).
- Reflection: mathematical thinking, generalization, abstraction and reflection, and complex mathematical problem solving" (p. 102)

Although research includes descriptions of formative assessment tasks, there are very few formative assessments that have been validated, and those that have validity information are primarily commercial products. The following table shows existing measures for children's mathematics learning. Berry, Bridges, Johnson, Calkins, Margie, Cochran, Ling and Zaslow (2008) provide additional information concerning internal, concurrent, and predictive validity can be found in by Berry and colleagues (2008). The following are not examples of formative assessments—as they are tests rather than assessment processes; however, we include them here because their content and tasks could be adapted for formative purposes.

Table 1: Early Childhood Measures

Measure	Subscale tested	Data Gathering & Type	Ages
Woodcock-Johnson III (WJ-III)	Broad Mathematics, cognitive abilities and test of achievement	Direct Assessment, Work Sampling Plans, Portfolio, Summative Instructional Tools (norm referenced) Also in Spanish	2 years to adulthood, although some subsets not appropriate for 2-4 year olds
Peabody Individual Achievement Test-Revised (PIAT-R)	Broad Mathematics, ranging from recognizing numbers to solving geometry and trigonometry problems	Direct Assessment, Work Sampling Plans, Portfolio, Summative Instructional Tools (norm referenced)	5 years to 18
Test of Early Mathematics Ability (TEMA)	Broad Mathematics	Direct Assessment, Work Sampling Plans, Portfolio, Summative Instructional Tools	3 to 8 years, 11 months
Stanford-Binet Intelligence Scale, Fourth ed. (SB-IV)	Quantitative Reasoning (Quantitative, number series, equation building)	Direct Assessment (norm referenced)	2 years through adulthood
Bracken Basic Concept Scale-Revised (BBCS-R)/SRC	Number/Counting Sizes, Shapes, Quantity	Direct Assessment (norm referenced) Also in Spanish.	2 years, 6 months to 8 years
Kaufman Assessment Battery for Children (K-ABC)	Sequential Processing Scale, Simultaneous Processing Scale, Mental Processing Scale, and Achievement scale	Direct Assessment (norm referenced)	2 years, 6 months to 12 years
Test of Early Mathematics Ability,	Formal (in school) and	Direct Assessment (norm referenced)	3 to 8 years, 11 months

Second ed (TEMA-2)	informal (out of school) learning in mathematics		
The Galileo System for the Electronic Management of Learning	Early Mathematics	Questionnaire and Observation	
High/Scope Child Observation Record (COR)	Logic and mathematics	Questionnaire and Observation	
The Work Sampling System (WSS)	Mathematical Thinking	Questionnaire and observation	PreK (age 3) to Grade 6

(Berry et al., 2008)

In developing a comprehensive mathematics formative assessment system and selecting the most suitable tasks for assessing mathematics learning, the critical competencies for success in mathematics need to be identified. A learning progression of core competencies necessary for successful mathematics learning should serve as a basis and a guide for creating assessments. The following is a general review of what foundational knowledge and understanding are most predictive of children's future mathematics learning and achievement. Describing each of the mathematic concepts and skills essential for mathematics learning and achievement is beyond the scope of this review. However, as part of the formative assessment project for PK-3 mathematics, the research team will construct learning progressions based on Florida state standards and the guidance of mathematics specialists. The following section introduces broader competencies critical for student growth in mathematics learning.

What foundational knowledge and understanding are most predictive of future mathematics learning and achievement?

<u>Cognitive science and assessment</u>

Advances in cognitive science research have changed our concept of learning (Pellegrino, Chudowsky, & Glaser, 2001) as well as our concept of what attributes lead to expertise in a domain, such as mathematics. How children store, organize, and synthesize information and how (and how quickly) they access prior knowledge reveal their readiness to advance to the next level of cognitive complexity and the next concept or skill to be mastered in the learning process for a domain. Understanding the contents of long-term memory and how well a child can retrieve knowledge stored there, incorporate new information, and apply reasoning to solve problems or create new learning is “especially critical for determining what people know; how they know it; and how they are able to use that knowledge to answer questions, solve problems, and engage in additional learning” (Pellegrino et al., 2001, p. 3). In addition to inspiring a reconceptualization of how learning takes place, cognitive science has also created a fundamental shift in what it means to understand subject matter. Subject matter proficiency is no longer based solely on what students *know*, but rather on “what students know and can *do* with their knowledge” (p. 59).

Although cognitive science has done much to advance understanding of what goes on in Black and Wiliam’s “black box” (1998), assessment techniques still need to be improved to make its content more visible and clear. To take advantage of cognitive research, assessments, particularly those included in classroom instruction, should focus on making students’ thinking visible to teachers as well as students, and “methods used in cognitive science to design tasks, observe and analyze cognition, and draw inferences about what a person knows are applicable to many of the challenges of designing effective educational assessments” (Pellegrino, 2001, p. 5). The Committee on the Foundation of Assessment recommends that when designing a classroom assessment “a model of cognition and learning” should serve as the organizing principle.

Studies of differences between expert and novice thinking reveal critical features of superior performance. Research indicates, for example, that experts in a domain can retrieve content knowledge and procedural information in chunks as opposed to individual pieces; consequently, experts seem to be able to process greater amounts of new and/or more complex information to solve problems more efficiently and accurately. Pellegrino, Chudowsky, and Glaser (2001) suggest that the attributes that lead to expert performance “should be the targets for assessments” (p. 4).

Designing comprehensive assessments that provide evidence of children’s earliest steps towards mathematics expertise may require changes in some of the concepts and visions of assessments and what they should measure to make the greatest impact on learning. Innovation to improve assessments could begin with a closer examination of students’ mathematics cognition—what strategies they use to translate knowledge into reasoning, into problem solving; what thoughts, beliefs, and even misconceptions lie within the "black box" (Black & Wiliam, 1998) of a child's mind. This evidence will enable teachers to address the origin of a student's mathematics failure as opposed to just correcting resulting errors.

Components of Mathematic Proficiency

General competencies of mathematics proficiency serve as guidelines for what we want children to accomplish so that they will be mathematically literate and able to navigate the mathematical situations they will confront as they expand their worlds. The specific components of mathematics proficiency and the steps learners take as they progress toward proficiency are not always specified completely (Gong, 2007). National Council of Teachers of Mathematics (NCTM), in *Principles and standards for school mathematics* (2000), lists the following as the content knowledge and processes children need to be proficient in mathematics: *Content*—number and operations, algebra, geometry, measurement, and data analysis and probability; Process—problem solving, reasoning and proof, making connections, oral and written communication, uses of mathematical representation (Schoenfeld, 2007a). In determining a student's mathematics proficiency—what a student "knows, can do, and is disposed to do mathematically"—Schoenfeld (2007b) believes four aspects of proficiency need to be considered: knowledge base, strategies, metacognition (using what you know effectively), and beliefs and dispositions. Proficiency involves not only knowing content and mastering procedures and skills but also understanding and comprehending the importance of the concepts and reasoning underlying mathematics work.

Kilpatrick, Swafford, and Findell (2001) identify five strands of mathematical proficiency:

(1) "Conceptual understanding: comprehension of mathematical concepts, operations, and relations
(2) Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
(3) Strategic competence: ability to formulate, represent, and solve mathematical problems

(4) Adaptive reasoning: a capacity got logical thought, reflection, explanation, and justification

(5) Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy." (Kilpatrick et al., 2001, p. 11)

They also contrast instructional approaches in countries consistently successful in producing students highly successful in mathematics to the approaches used in the United States (National Academy of Sciences, 2001). Other countries select a few crucial topics to cover in each grade and teach the topic in depth and to cognitively complex level. The teachers stay with each topic until students master the concepts and procedures related to the topic. In the following year, they do not return to the previous topics but select new, more advanced topics. In the U. S. curriculum, topics are not considered to be the province of one grade, to be mastered and not returned to. Instead instruction of a topic often spans several years; each year a level of complexity regarding the same concept is added (National Academy of Sciences, 2001). Appendix A contains tables comparing the U. S. and other countries regarding what mathematics topics are covered and at what grade-levels.

The ability to identify facts, repeat definitions, and perform procedures is not enough; students need to have the depth of understanding and dexterity in reasoning to use their mathematical knowledge and skill to create problem-solving strategies. Proficient students need to be able to formulate problems; to represent them through mathematical language, notations, and models; and to synthesize their resources to solve the problem. Students need to recognize and avoid the pitfall of becoming so focused on carrying out steps or procedures and computations that they don't reflect on what they are doing and whether or not their process is advancing them towards their goals. In a classic "forest-for-the trees" dilemma, students who don't monitor their progress and self-regulate their problem-solving strategies as they are working to solve a problem may work through a calculation or solve an equation only to find they have not solved the relevant problem. Mathematical problem solving, like most problem-solving domains, requires the ability to analyze and assess the validity and relevance of one's thinking.

As both NCTM (2000) and Schoenfeld (2007) point out, proficiency in mathematics involves other competencies, such as communication skills, reading and listening comprehension (especially for solving word problems), and a belief in the relevance and logic of mathematics in school and in the world outside of school. Negative beliefs that could undermine students' interest and success in mathematics include (a) math problems have only one correct answer, and there is only one way to arrive at that answer; (b) ordinary students may be able to memorize and apply formulas but they cannot really *understand* math; and (c) explaining how you arrived at an answer or proving its correctness are not relevant.

Achieving mathematical proficiency is an important and challenging goal for students and adults. However, it seems that more than proficiency is needed to grow the pool of talented mathematicians who will develop and lead innovation needed to renew our economy and global competitiveness. Young children need a strong and reliable foundation of mathematical skills and understanding to support and scaffold future and advanced mathematics learning.

Most young children have the ability to "solve problems involving simple mathematical reasoning by age 3" (NRC, 2008, p. 116). Furthermore, even first and second graders can make the mathematical generalizations and reasoning about them that form the foundation for learning and understanding algebra (Carpenter, Levi, & Farnsworth, 2000, p. 3). Since the foundations for mathematical learning begin to form so early, the need for early interventions that prevent the formation of misconceptions and instill correct understanding are important. Effective research-based preschool mathematics curriculum is needed. Clements and Sarama (2008) conducted randomized trials to evaluate the preschool curriculum called Building Blocks and found that the scores of children in the intervention group significantly increased. One of the characteristic of the Building Blocks program is that the teachers were more involved with the students during their activities and used "formative assessment based on their knowledge of children's developmental progressions" (Clements & Sarama, 2008, p. 486). The success of the Building Block teachers suggests the positive contribution learning trajectories (or progressions) make to learning, and the researchers posit that the "instructional strategies and pedagogical content knowledge embedded in its learning trajectories," may account for the Building Block success (p. 489). Clement and Sarama provide tables showing the assessment items used to test the students in the experimental groups and how each group responded.

Research suggests that number sense, particularly counting, is one of the earliest stages of mathematical development (Purpura, 2008). Children begin learning formal mathematics skills—such as writing numerals and using abstract numerical notation—after they enter school, but they develop informal skills and concepts, and possibly misconceptions and faulty knowledge, as they explore their early environments. Accurate knowledge and understanding serve as a foundation for formal mathematics learning. Children may arrive at school lacking the informal skills, the very basic foundation to support beginning and future formal learning. Children's early exposure to a rich environment that stimulates cognitive growth and mathematics learning influences their readiness to begin learning mathematics in school and often their long-term mathematics learning. "[C]hildren who start kindergarten behind their peers tend to stay behind throughout their schooling" (National Mathematics Advisory Panel, 2008, p. 4-xiii).

Students from families with a single-parent, low-parental education, for whom English is a second language, and those living in poverty often lack prerequisite informal skills and tend to be at greater risk for low achievement in mathematics. Young children need to be assessed early to determine where they are in their development in order to target early instruction at correcting or establishing a strong foundation that can support years of additional mathematics learning. Assessments, however, do not agree on what informal or formal skills are core skills predictive of readiness to learn mathematics (Purpura, 2008). Although no study has "examined the developmental trajectories from specific informal number-related skills in preschool" to the development of later mathematical skills, some evidence indicates that "measures of quantity discrimination, identification of missing numbers, and knowledge of magnitude comparisons are all reasonable correlates of later mathematics learning" (Purpura, 2008, p. 10). For descriptions of informal mathematical skills and means for assessing these skills, see Appendix B.

According to the National Mathematics Advisory Panel (NMAP), mathematical understanding in preschool includes acquisition of number words and counting, ordering numbers, arithmetic, measurement, geometric knowledge, and number sense. Number word and counting competencies include counting objects, understanding principles underlying counting procedures; and ordering numbers includes understanding the relative magnitude of numbers. Preschool-age children begin doing simple arithmetic, such as basic addition and subtraction, and they develop counting strategies (e.g., counting on fingers). They also begin understanding some arithmetic concepts; for example, addition and subtraction are inverse operations. Early development of measurement skills and understanding include simplistic notions of relations of equality, less than, and more than. Geometric knowledge begins with identifying circles, squares, and triangles and then discriminating between squares and rectangles and developing basic spatial knowledge. As they develop number sense they begin to approximate numerical magnitudes. NMAP emphasizes the need to make visible and understand the cognitive processes supporting such operations (NMAP, 2008).

Panel members concluded that the large body of high-quality research in cognitive science and the information it provides about the processes that support mathematics learning are not being adequately used. This conclusion suggests the need for high-quality formative assessment tasks that provide evidence of children's thinking processes as they solve mathematical problems. Such tasks would empower teachers to target instruction to correct cognitive errors or weaknesses that undermine the child's ability to grasp the content being taught and limit or prevent future learning.

Algebra is considered the gateway to advanced mathematics, so the skills and understanding students need to enter and succeed in algebra are often considered fundamental to advanced math. Therefore, the National Mathematics Advisory Panel focused research on effective student preparation for and success in algebra (NMAP, 2008). The NMAP concluded, "To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem-solving skills" in three foundational areas: fluency with whole numbers, fluency with fractions, and particular aspects of geometry and measurement (NMAP, 2008, p. 3-xi). Fluency with whole numbers should be achieved by the end of elementary school.

Counting ability is a powerful predictor of mathematics development in young children (Aunola, Leskinin, Lerkkanen, & Nurmi, 2004). Aunola et al. (2004), in their investigation of “the developmental dynamics of mathematical performance during children’s transition from preschool to Grade 2” and the cognitive antecedents that support this development (p. 699), found that children’s counting ability predicted both their initial mathematics performance level and their growth. Furthermore, “children with math-related disabilities are deficient in their basic counting abilities regardless of their IQ or reading status” (p. 700). One reason why counting may be so essential to future math learning is that counting needs to become automatic to free up working memory to attend to increasingly complex problems.

As part of their study, Aunola et al. (2004) used the Diagnostic Test for Basic Mathematical Concepts (Ikaheimo, 1996) to assess students’ mathematical development across the following measurement points: (a) knowledge of ordinal numbers, (b) knowledge of cardinal numbers and basic mathematical concepts, (c) number identification, (d) word problems, and (e) basic arithmetic. The results of the assessments suggested other cognitive antecedents to mathematics learning.

Metacognitive knowledge: Children’s “knowledge and understanding of cognitive processes” and their ability to reflect on and evaluate their own thinking as they solve problems. Aunola et al. (2004) cite two possible reasons for why metacognition is so important to math development:

- “metacognitive ability may reflect overall cognitive ability,… which has been shown to be highly correlated with academic achievement in mathematics
- metacognitions may enable learners to adjust to varying problem-solving tasks, demands, and contexts” (p. 700).

The study found that “the higher the level of metacognitive knowledge at the beginning of the preschool year, the higher the initial level of math performance”; however it was not predictive of growth. One possibility is that metacognitive skills may help children utilize existing knowledge but not be as important in acquiring new knowledge.

Attentional resources: Young children with attention-deficit/hyperactivity disorder demonstrate “delayed automatization of number facts” so critical to children’s progress in mathematics (p. 701). The study found that visual attention predicted rate of growth in math but not initial performance. Researchers suggest that as children perform increasingly complex tasks, greater attention and working memory are needed.

Listening comprehension: Listening comprehension is another possible antecedent to successful mathematical learning because of its importance in handling verbally presented mathematical problems. The study found that like metacognitive skills it seems to be more important for supporting the use of existing knowledge than contributing to acquisition of new knowledge.

The findings on cognitive antecedent suggest that to improve instruction and assessment (a) automatization of basic math facts requires “drill and practice in addition to constructivist-based instruction” (p. 710), and (b) a focus on skills other than just mathematics skills, such as metacognition and attention, is needed to improve mathematics learning.

Young children’s competencies needed for future mathematics learning

In *Early Childhood Assessment: Why, What, and How* (2008), the National Research Council’s Committee on Developmental Outcomes and Assessments for Young Children explains that as early as infancy, children begin acquiring knowledge of the foundations of future mathematics learning: “number sense, spatial sense and reasoning (geometry), measurement, classification and patterning (algebra), and mathematical reasoning” (NRC, 2008, p. 114).

Number sense: Research indicates that “children begin developing number sense in infancy…and much of what young children know about numbers depends on their understanding and mastery of counting” (NRC, 2008, p. 114). Mastery of counting requires three fundamental skills: “knowing the sequence of number words, one-to-one correspondence, and cardinality” (NRC, 2008, p.114). Once a child can count, he or she begins to understand number operations and then simple operations and word problems. Number operations for preschoolers primarily involve understanding that two or more smaller numbers make up one larger number. This understanding is the prerequisite for addition and subtraction concepts in the future.

Geometry: In addition to recognizing and naming shapes, children also need to understand the characteristics and properties of shapes. Researchers believe that children “learn about geometry on a progression of levels—visualization, analysis, abstraction, deduction, and rigor” (NRC, 2008, p. 115). Children’s spatial reasoning involves being able to identify objects and their location, direction, and distance.

Measurement: Children must be able to identify characteristics of an object that can be measured, such as length or width. They also need to recognize the appropriate unit of measures to use when measuring the object, such as inches or pounds, and to "use measuring skills and tools to compare the units" (NRC, 2008, p.115). A typical progression of measurement knowledge and understanding, involves:

1. "Learning to use words that represent quantities or magnitude of a certain attribute (e.g., big and small)
2. Demonstrating an ability to compare two objects directly and recognize equality and inequality
3. Learning to measure, connecting numbers to attributes of objects, such as length, weight, amount, area, and time" (NRC, 2008, p. 115)

Algebra: Young children begin to develop algebraic concepts "as they sort and classify objects, observe patterns in their environment, and begin to predict what comes next based on a recognized pattern. These concepts help children order their world and establish some degree of predictability about the future. "Classification and the analysis of patterns provide a foundation for algebraic thinking as children develop the ability to recognize relationships, form generalizations, and see the connections between common underlying structures" (NRC, 2008, p.115). Classification involves categorizing objects, arranging them, and then grouping those objects based on established criteria. Understanding patterns is critical to mathematics learning as well as learning in other domains. To understand a pattern, children "identify similarities and differences among the elements of the pattern, note the number of elements in the repeatable group, identify when the first group of elements begins to replicate itself, and make predictions about the order of elements based on given information" (NRC, 2008, p.115). Research suggests that the ability to perform these skills depends on the child's ability to identify the "core unit of the pattern," and such identification often depends on a child's experiences at home as well as educational settings.

Mathematical reasoning: Three conditions support young children's development of mathematical reasoning: "children must have a sufficient knowledge base; the task must be understandable and motivating; and the context of the task must be familiar and comfortable to the problem solver" (NRC, 2008, p. 116).

Conclusion

This review of literature on formative assessment reveals the need for additional high-quality, empirical research on the impact formative assessment systems have on student learning, particularly in the areas of mathematics and early mathematics learning. Research may be enhanced if studies examined separately the validity and reliability of each component of formative assessment (e.g., tasks and feedback). Existing literature does demonstrate the value of evidenced-based instruction and task-level, classroom-based assessments.

The information gained from the review of literature has assisted the project team in identifying crucial tasks in the development of a formative assessment system for PK-3 mathematics. Essential tasks are (a) the development of comprehensive and detailed learning progressions based on the Florida state standards and input from mathematics specialists, (b) the creation of assessment items and tasks that will provide evidence of student learning in these crucial areas of mathematics concept and skill development, and (c) the creation of feedback protocols for a range of classroom environments and student populations.

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Appendix A

Grade-Level Expectations for Mathematics

Source:
National Mathematics Advisory Panel. (2008). *Foundations for Success: Reports of the Task Groups and Subcommittees of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

Table 6: K Through 8 Grade-Level Expectations in the Six Highest-Rated State Curriculum Frameworks in Mathematics Compared With the Topics Intended by a Majority of TIMSS 1995 Top-Performing Countries*

		Grade								
Topics		**K**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	Whole Number Meaning	**6**	**5**	**5**	**4**	**6**	**2**			
2	Whole Number Operations	**3**	**4**	**5**	**6**	**3**				
3	Measuring and Units of Measurement	**6**	**6**	**2**	**3**	**4**	**2**	**6**		
4	Common Fractions	**3**	**1**	**4**	**4**	**5**	**3**			
5	Equations and Formulas				**2**	**2**	**2**	**4**	**6**	**6**
6	Collecting, Evaluating, Interpreting, and Representing Data	**4**	**5**	**5**	**3**	**4**	**5**	**5**	**3**	**4**
7	2-D Geometry Basics		**4**	**5**	**5**	**4**	**2**	**3**		
8	Polygons and Circles	**6**	**6**		**1**	**1**	**2**	**2**		**1**
9	Perimeter, Area, and Volume				**2**	**2**	**5**	**4**	**5**	**4**
10	Rounding				**4**	**5**	**4**			
11	Estimating Computations and Determine Reasonableness				**3**	**3**	**3**	**3**	**1**	
12	Properties of Whole Number Operations				**5**	**2**	**1**	**3**	**5**	**4**
13	Decimal Fractions and Decimals			**6**	**2**	**4**	**1**	**1**	**2**	**4**
14	Percentages						**5**	**4**	**2**	**1**
15	Ratios and Proportions							**2**	**1**	**2**
16	2-D Coordinate Geometry				**3**	**4**	**2**	**4**	**2**	
17	Geometric Transformation				**1**	**3**	**1**	**1**	**3**	**3**
18	Integers & Their Properties						**4**	**4**		
19	Number Theory						**5**	**3**	**1**	**1**
20	Exponents, Roots, Radicals, and Absolute Values							**3**	**5**	**6**
21	3-D Geometry				**6**	**2**	**2**	**1**	**3**	**1**
22	Congruence and Similarity		**1**	**2**	**2**	**5**	**2**	**2**	**4**	**3**
23	Rational Numbers and Their Properties							**2**	**6**	**5**
24	Patterns	**6**	**3**	**3**	**3**	**3**	**4**	**4**	**4**	**2**
25	Functions and Relations							**2**	**2**	**4**
26	Slope and Rates of Change							**1**	**2**	**3**
27	Probability		**1**	**3**	**3**	**3**	**3**	**5**	**1**	**4**

Note: A shaded cell indicates that at least one state specified an objective in this area at the given grade level.

STPI used 30 topics for its original analysis of these six state frameworks. Twenty-five of these topics agree with the TIMSS topics in Figure 7. Three of the remaining five topics were eliminated chiefly because they appeared to overlap with existing TIMSS topics. It is important to note that how STPI defined its 30 topics may differ from how these topics were defined by Schmidt et al. (2002) since it is not completely clear from the latter's writings how the 30 topics in Figure 7 were defined. A shaded cell indicates that at least one state specified an objective in this area at the given grade level.

The numbers in the shaded cells refer to the number of states within the six states that list that topic in their curriculum expectations.

Source: Institute for Defense Analyses Science and Technology Policy Institute, in press, b.

Table 6: K Through 8 Grade-Level Expectations in the Six Highest-Rated State Curriculum Frameworks in Mathematics Compared With the Topics Intended by a Majority of TIMSS 1995 Top-Performing Countries*

	Topics	Grade								
		K	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	Whole Number Meaning	**6**	**5**	**5**	**4**	**6**	**2**			
2	Whole Number Operations	**3**	**4**	**5**	**6**	**3**				
3	Measuring and Units of Measurement	**6**	**6**	**2**	**3**	**4**	**2**	**6**		
4	Common Fractions	**3**	**1**	**4**	**4**	**5**	**3**			
5	Equations and Formulas				**2**	**2**	**2**	**4**	**6**	**6**
6	Collecting, Evaluating, Interpreting, and Representing Data	**4**	**5**	**5**	**3**	**4**	**5**	**5**	**3**	**4**
7	2-D Geometry Basics		**4**	**5**	**5**	**4**	**2**	**3**		
8	Polygons and Circles	**6**	**6**		**1**	**1**	**2**	**2**		**1**
9	Perimeter, Area, and Volume				**2**	**2**	**5**	**4**	**5**	**4**
10	Rounding				**4**	**5**	**4**			
11	Estimating Computations and Determine Reasonableness				**3**	**3**	**3**	**3**	**1**	
12	Properties of Whole Number Operations				**5**	**2**	**1**	**3**	**5**	**4**
13	Decimal Fractions and Decimals			**6**	**2**	**4**	**1**	**1**	**2**	**4**
14	Percentages						**5**	**4**	**2**	**1**
15	Ratios and Proportions							**2**	**1**	**2**
16	2-D Coordinate Geometry				**3**	**4**	**2**	**4**	**2**	
17	Geometric Transformation				**1**	**3**	**1**	**1**	**3**	**3**
18	Integers & Their Properties						**4**	**4**		
19	Number Theory						**5**	**3**	**1**	**1**
20	Exponents, Roots, Radicals, and Absolute Values							**3**	**5**	**6**
21	3-D Geometry				**6**	**2**	**2**	**1**	**3**	**1**
22	Congruence and Similarity		**1**	**2**	**2**	**5**	**2**	**2**	**4**	**3**
23	Rational Numbers and Their Properties							**2**	**6**	**5**
24	Patterns	**6**	**3**	**3**	**3**	**3**	**4**	**4**	**4**	**2**
25	Functions and Relations							**2**	**2**	**4**
26	Slope and Rates of Change							**1**	**2**	**3**
27	Probability		**1**	**3**	**3**	**3**	**3**	**5**	**1**	**4**

Note: A shaded cell indicates that at least one state specified an objective in this area at the given grade level.

STPI used 30 topics for its original analysis of these six state frameworks. Twenty-five of these topics agree with the TIMSS topics in Figure 7. Three of the remaining five topics were eliminated chiefly because they appeared to overlap with existing TIMSS topics. It is important to note that how STPI defined its 30 topics may differ from how these topics were defined by Schmidt et al. (2002) since it is not completely clear from the latter's writings how the 30 topics in Figure 7 were defined. A shaded cell indicates that at least one state specified an objective in this area at the given grade level.

The numbers in the shaded cells refer to the number of states within the six states that list that topic in their curriculum expectations.

Source: Institute for Defense Analyses Science and Technology Policy Institute, in press, b.

Appendix B

Assessing Informal Mathematical Skills

Source:

Purpura, D. J. (2008). *Informal number-related mathematics skills: An examination of the structure of and relations between these skills in preschool*. Unpublished doctoral dissertation, Florida State University, Tallahassee.

Counting backwards	Higher order counting ability that indicates a child's knowledge that the counting sequence can be used for different purposes than simply counting a set.	REMA: Children are asked to start counting backwards from a specified number (e.g. 10). ENT: Children are presented with a specific number of blocks, told how many blocks there are, and instructed to point out the blocks and count backwards. TEMA-3: Children are asked to count backwards from 10.
Counting error identification	Knowledge of the principles of counting (e.g. a number is counted only once, numbers cannot be skipped, each object counted is assigned only one number) and ability to recognize when counting principles have been violates.	CMA: Examiner counts a series of objects either correctly, incorrectly (repeats a number, skips an object, double counts an object, etc), or pseudo-incorrectly (counts every other object, then goes back through and counts the missing dots). Then asks the child to indicate whether the objects were counted correctly. NSC: Children were asked to identify whether the examiner counted "OK" or "not OK." The examiner then counted the set of objects either correctly, pseudo-incorrectly (e.g. counting every other dot then going back through and counting the missed dots), or incorrectly (e.g. skipping a picture, double-counting a picture).
Structured Counting	The ability to utilize the counting sequence to enumerate a quantity.	REMA: Children are shown a set of pictures and asked to count them and tell the examiner how many there are. CMA: Children were asked to count a set of objects. NSC: Children were asked to count a set of pictures (e.g. count a set of 5) ENT: A number of blocks are placed in front of the child. They are instructed to count the blocks. Pointing, touching, and moving the blocks is allowed. NKT: Children are asked to count a specific number of objects presented to them. TEMA-3: Children are asked to count a specific number of pictures.

Cardinality	Recognition that the last number counted means “how many.”	REMA: Children were asked to count a set of objects and then to specificy how many there are. NSC: Children were asked to count a set of pictures out loud. The pictures were then hidden from view and children were asked, “How many of the picture was on the paper. TEMA-3: Child is presented with a number of pictures. The pictures are then hidden and the child is asked, “How many [of the picture] did you count?”
Resultative Counting	Higher order counting skill that indicates a child’s ability to count without physical manipulation or touching of the set to be enumerated.	ENT: Children are presented with a specific number of blocks and asked to identify how many total objects there are. Pointing, touching, and moving the blocks is NOT allowed.
Count a subset of objects	The understanding that quantities can be separated from a larger quantity.	REMA: Children are instructed to produce a set of a specific number out of a larger set of objects. CMA: Child is asked to count all the chips of one color out of a set with two different colored objects ENT: Child is presented with a set of blocks (e.g. 15). They are then asked to produce a smaller set of the blocks (e.g. 11). NKT: Children are presented with a line of objects of two different colors. Then the child is asked to count all of one color object. TEMA-3: Children are given a set of blocks. They are then instructed to give the examiner a subset of the objects.
Subitizing	The ability to rapidly enumerate small quantities without counting the set item by item.	REMA: Children are shown a card with a specific number of the same object on it for exactly 2 seconds. The card is then hidden and the child is asked, “How many?”
Estimation	Ability to quickly understand the approximate quantity in a moderate to large set without structured counting.	NSC: Children were presented with cards with a specific number of dots (ranging from 3 to 35). Children were asked, “About how many dots do you see.” Responses were scored correct if the child provided a response within 25% of the correct answer.

REMA = Research-based Early Math Assessment (Clements, Sarama, & Liu, 2008), CMA = Child Math Assessment, NSC = Number Sense Core battery (Jordan et al., 2007), ENT = Early Numeracy Test (van de Rijt, van Luit, & Pennings, 2003), NKT = Number Knowledge Test (Griffin & Case, 1997), TEMA-3 = Test of Early Mathematics Ability – Third Edition (Ginsburg & Baroody, 2003).

Table 2

Names, descriptions, and means of assessing Numerical Relations informal number skills

Skill	Description	Assessment Examples
Ordinal Numbers	Identification of ordinal positions such as 1st, 2nd and 3rd, and last.	REMA: “Three animals are in a line, which one is first in line.” CMA: Child is asked to identify the object in a specific ordinal position (1st, 2nd, etc). ENT: “Point out the Nth object.”
Relative Size	Knowledge of the proximity of one number to another number along the number line.	REMA: Children are asked which of two numbers is closer to another number (e.g. which of 500 and 520 is closer to 250). NSC: Children were shown three numbers (e.g. 4, 7, 8) and were asked which number (e.g. 7 or 8) is closer to the other number (e.g. 4). NKT: Children are asked which of two numbers is closer to another number. The visual array of numbers is then shown to the child. TEMA-3: Children are presented an array of three numbers and asked which of two of the numbers is closer to the other number.
Number Comparison	The ability to differentiate the numerical quantity of two numbers.	REMA: Children were verbally asked which of two or three numbers is bigg[er/est] or small[er/est]. CMA: Children are asked to identify the larger or smaller of two numbers. NSC: Children were verbally given two numbers (e.g. 4 and 5) and asked which number is bigger or which number is smaller. NKT: Children are verbally asked which of two numbers is bigger or smaller. TEMA-3: Children are verbally asked questions such as, “Which is more, 3 or 2?”

Set Comparison	Ability to differentiate magnitude of sets of varying sizes.	REMA: Child is shown two cards with different quantities of dots and is asked to identify which card has more dots. Children are also shown cards with dots of different sizes (e.g. 9 big dots on one card and 11 small dots on another card) and asked which is more. ENT: Which set of four sets of objects has the fewest? NKT: Children are presented with two sets of objects and asked to identify the pile with either more or less objects. TEMA-3: Child is asked to identify which of two sets of dots has more dots.
Number Order	Knowledge of the ordering of numbers on the number line by identifying the numbers before and after a given number.	REMA: Child is verbally asked to identify which number comes one or two before or after a specified number. CMA: Child is verbally asked to identify the number that come before or after another number (e.g. What number comes after 4?) NSC: Children were verbally given a number (e.g. 4) and asked what number comes after that number and what number comes two after that number. NKT: Child is asked, “What number comes right after 7?” and “What number comes 2 after 7?”
Sequencing	Knowledge of the order of numbers and quantities along the number line.	REMA: Children are asked to correctly order a set of six cards by the number of images on a card. Children are also asked to order a set of 1-5 numerals. CMA: Children are asked to order a set of five objects in a series by size and insert a sixth object. This task does not use numbers or quantities but, rather, uses physical size. ENT: Children are presented with several or the same object in varying quantity. They are also presented with another example of the same object and asked where it goes in the order based on the quantity.

Set reproduction	Ability to produce or recognize equivalent sets by using knowledge of one-to-one correspondence.	REMA: Children are shown a picture of a specific number of objects. They are then given objects and asked, "Make yours look just like mine." CMA: Children are presented with a number of objects of one color and asked to produce the same number of that object with another color. ENT: Child is visually presented with a number of objects. They are then given a larger number of blocks and asked, "Can you lay down the same amount of blocks?" Children are also asked to match equal sets of different types of objects (e.g. draw lines from the pictures that have the same number of objects). TEMA-3: Children are presented with a number of objects. The number of objects is then hidden and the child is asked to reproduce the same number of blocks.
Number ID	Knowledge of the names of written numerals.	NSC: Children were shown a series of cards, each with a given number (e.g. 3) and asked to verbally name the number. TEMA-3: Child is visually presented with a numeral and asked, "What number is this?"
Numerals	Ability to connect quantity to written numbers.	REMA: Children were asked to match a numeral to a set of objects of the same number. TEMA-3: Children are shown pictures of animals and asked to write down the number of animals on the page.

REMA = Research-based Early Math Assessment (Clements, Sarama, & Liu, 2008), CMA = Child Math Assessment, NSC = Number Sense Core battery (Jordan et al., 2007), ENT = Early Numeracy Test (van de Rijt, van Luit, & Pennings, 2003), NKT = Number Knowledge Test (Griffin & Case, 1997), TEMA-3 = Test of Early Mathematics Ability – Third Edition (Ginsburg & Baroody, 2003).

Table 3

Names, descriptions, and means of assessing Arithmetic Reasoning informal number skills

Skill	Description	Assessment Examples
Addition/Subtraction with objects	Ability to identify specific changes in magnitude of a set of objects.	REMA: Children are given a specific number of objects (e.g. 3) and told how many objects they were given. They were then given another quantity of the same object (e.g. 2) and told how many more of that object they received. They were then asked how many they were given all together. CMA: A number of objects are placed in front of the child. The child is then asked to determine the total number of objects on the table. The objects are put in a box and told that another amount of objects are being added to the box. The child is asked to determine the total number of objects in the box. Similar procedures are done for subtraction. NSC: The experimenter placed X objects on the table and told the child how many objects were there were. The objects were then covered and objects were either added or subtracted one at a time. The child was told how many objects were being added or subtracted. The objects remained covered and the children were asked how many total objects were now on the table. ENT: Children are shown a picture of a specific number of objects (e.g. 9 marbles). They are then told that another number of objects (e.g. 3) is lost. They are then asked to identify the picture at the bottom of the page with the total number of objects that are left. TEMA-3: Examiner places block(s) on the table, then covers the block(s) on the table. The examiner then places more blocks on the table and proceeds to hide them with the other blocks. The child is then instructed to “Make [theirs] the same as [the examiners].” Children are also presented with story problems and instructed that they can use their fingers or blocks to help solve the problems.

Addition/Subtraction without objects	Ability to solve simple addition and subtraction story problems apart from the formal written structure.	CMA: Story problems were verbally presented to children such. NSC: Story problems were verbally presented to children such as "Jill has two pennies. Jim gives her one more penny. How many pennies does Jill have now?" NKT: Story problem, If you had 4 chocolates and I gave you 3 more chocolates, how many would you have? TEMA-3: Children are presented verbally with story problems.
Equivalent Sets	Ability to equally divide a given number of objects between a different number of groups and if necessary recognize that the set cannot be divided equally.	CMA: Children are asked to divide a number of objects amongst a different number of individuals (e.g. equally divide these 6 objects among these 3 people). TEMA-3: Children are given a number of blocks and asked to share the blocks fairly between two people.
Two-set addition	Ability to recognize that changes in quantity to one set of objects results in a change to that set's quantitative relation to another set.	CMA: Objects are added to two separate containers, one at a time (e.g. one in container one, one in container two, another in container 1, etc). The child is then asked to determine if the containers have the same number of objects or if one has more objects (child cannot see inside the containers). One or two more objects are then either added to or subtracted from one of the containers and the child are again asked to determine if the containers have the same number of objects or which has more.
Number composition/decomposition	Ability to determine changes in initial quantities by knowing initial and final quantities.	REMA: Examiner places a specific number of objects (e.g. 4) in front of the child and counts them out. Examiner then covers the object and either adds or subtracts a specific number of objects. Child is then shown the total number of objects and asked to identify how many objects were either added or subtracted.

Number Combinations	Ability to solve basic addition and subtraction problems such as 1+1=2, presented verbally and/or visually.	REMA: Children were verbally asked questions such as “How much is 2+7?” NSC: Children were verbally asked addition and subtraction problems such as, “How much is 2 plus 1?” or, “How much is 2 minus 1?” The same problems were asked in this task as were asked in the verbal and non-verbal problem solving groups. NKT: Children are verbally asked questions such as, “How much is 2 + 4?” or, “How much is 8 take away 6?” TEMA-3: Children are visually shown addition and subtraction problems. The problems are then read to the child and the child is asked to solve the problem.

REMA = Research-based Early Math Assessment (Clements, Sarama, & Liu, 2008), CMA = Child Math Assessment, NSC = Number Sense Core battery (Jordan et al., 2007), ENT = Early Numeracy Test (van de Rijt, van Luit, & Pennings, 2003), NKT = Number Knowledge Test (Griffin & Case, 1997), TEMA-3 = Test of Early Mathematics Ability – Third Edition (Ginsburg & Baroody, 2003

Table 4

Listing of the important Counting, Numerical Relation, and Arithmetic Reasoning skills on five measures of informal mathematics skills and the National Council of Teachers of Mathematics standards.

	Measures						
Tasks	NCTM	REMA	CMA	NSC	ENT	NKT	TEMA-3
Counting							
Rote Counting		X	X	X	X	X	X
Counting up		X			X		X
Counting backwards		X			X		X
Counting error identification		X	X	X			
Structured Counting	X	X	X	X	X	X	X
Cardinality		X		X			X
Resultative Counting					X		
Counting Subsets		X	X		X	X	X
Subitizing	X	X					
Estimation				X			
Numerical Relations							
Ordinal Numbers		X	X		X		
Relative Size		X		X		X	X
Number Comparison	X	X	X	X		X	X
Set Comparison	X	X			X	X	X
Number Order		X	X	X		X	
Sequencing	X	X	X		X		
Set Reproduction		X	X		X		X
Number Identification				X			X
Numerals	X	X					X
Arithmetic Reasoning							
Addition/Subtraction with objects		X	X	X	X		X
Addition/Subtraction without objects			X	X		X	X
Two-set addition			X				
Number Composition		X					
Equivalent Sets			X				X
Number Combinations		X		X		X	X

NCTM = National Council of Teachers of Mathematics Standards, REMA = Research-based Early Math Assessment (Clements, Sarama, & Liu, 2008), CMA = Child Math Assessment, NSC = Number Sense Core battery (Jordan et al., 2007), ENT = Early Numeracy Test (van de Rijt, van Luit, & Pennings, 2003), NKT = Number Knowledge Test (Griffin & Case, 1997), TEMA-3 = Test of Early Mathematics Ability – Third Edition (Ginsburg & Baroody, 2003).

Appendix C

Examples of Learning Progressions

Source:

Heritage, M. (2008). *Learning progressions: Supporting instruction and formative assessment*. Washington, DC: Council of Chief State School Officers.

APPENDIX

Examples of Learning Progressions

A Counting and Ordering Progress Map

(Australian mathematics profile)

From Curriculum Corporation. (1994). *Mathematics Profile for Australian Schools.* Carlton: Curriculum Corporation. In Masters, G., & Forster, M. (1997). *Developmental Assessment*. Victoria, AU: The Australian Council for Educational Research Ltd.

5

Uses unitary ratios of the form 1 art to X parts
 (The ratio of cordial to water was 1 to 4)
Understands that common fractions are used to describe ratios of parts to whole
 (2 in 5 students ride to school. In school of 550, 220 ride bikes)
Uses percentages to make straightforward comparisons
 (26 balls from 50 tries is 52%; 24 from 40 tries is 60%, so that is better)
Uses common equivalences between decimals, fractions and percentages
 ('One-third off is better than 30% discount')
Uses whole number powers and square roots in describing things
 (finds length of side of square of area 225 sq cm as square root of 225)

4

Counts in decimal fraction amounts ('0.3, 0.6, 0.9, 1.2, …')
Compares and orders decimal fractions
 (orders given weight data for babies to two decimal places)
Uses place value to explain the order of decimal fractions
 (which library book comes first 65.6 or 65.126? why?)
Reads scales calibrated in multiples of ten
 (reads 3.97 on a tape measure marked in hundredths, labeled in tenths)
Uses the symbols =, < and > to order numbers and make comparisons
 (6.75 < 6.9; 5 x $6 > 5 x $5.95)
Compares and orders fractions (one-quarter is less than three-eights)

3

Counts in common fractional amounts
 ('two and one-third, two and two-thirds, three, three and one-third')
Uses decimal notation to two places
 (uses 1.25m for m 25cm; $3.05 for three $1 coins and one 5c coin; 1.75kg for 1750g)
Regroups money to fewest possible notes and coins
 (11x $5 + 17x $2 + 8 x $1 regrouped as 1x $50 + 2x $20 + $5 +$2)
Uses materials and diagrams to represent fractional amounts
 (folds tape into five equal parts, shades 3 parts to show 3/5)
Expresses generalizations about fractional numbers symbolically
 ('1 quarter = 2 eighths' and '1/4 = 2/8')

2

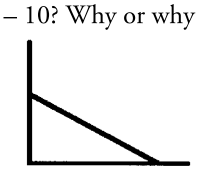
Counts forwards and backwards from any whole number, including skip counting in 2s, 3s, and 10s
Uses place value to distinguish and order whole numbers
 (writes four ten dollar notes and three one dollar coins as $43)
Estimates the size of a collection
 (up to about 20)
Uses fractional language (one-half, third, quarter, fifth, tenth) appropriately in describing and comparing things
Shows and compares unit fractions
 (finds a third of a cup of sugar)
Describes and records simple fractional equivalents
 ('The left over half pizza was as much as two quarters put together')

1

Counts collections of objects to answer the question 'How many are there?'
Makes or draws collections of a given size
 (responds correctly to 'Give me 6 bears')
Makes sensible estimates of the size of small collections up to 10
 (for 7 buttons, 2 or 15 would not be a sensible estimate, but 5 would be)
Skip counts in 2s or 3s using a number line, hundred chart, or mental counting
 ('2, 4, 6 …')
Uses numbers to decide which is bigger, smaller, same size
 (if he has 7 mice at home and I have 5, then he has more)
Uses the terms first, second, third ('I finished my lunch second')

A Developmental Model for Learning Functions

From National Research Council of the National Academies. (2005). How Students Learn: History, Mathematics, and Science in the Classroom. Washington, D.C.: The National Academies Press.

Level	General Description	Example Tasks & Understandings
0	Students have separate numeric and spatial understandings. • Initial numeric understanding: Students iteratively compute (e.g., "add 4") *within* a string of positive whole numbers. • Initial spatial understanding: students represent the relative sizes of quantities as bars on a graph and perceive patterns of qualitative changes in amount by a left-to-right visual scan of the graph, but cannot quantify those changes.	Extend the pattern 3, 7, 11, 15, __, __, __. Notice in a bar graph of yearly population figures that each bar is taller than the previous bar.
1	Spatial and numeric understandings are elaborated and integrated, forming a central conceptual structure. • Elaboration of numeric understanding: --Iteratively apply a single operation to, rather than within, a string of numbers to generate a second string of numbers. --Construct an algebraic expression for this repeated operation. • Elaboration of spatial understanding: --Use continuous quantities along the horizontal axis. --Perceive emergent properties, such as linear or increasing, in the shape of the line drawn between points. • Integration of elaborated understandings: --See the relationship between the differences in the y-column in a table and the size of the step from one point to the next in the associated graph. • Interpret algebraic representations both numerically and spatially.	Multiply each number in the sequence: 0, 1, 2, . . . by 2 to get a set of pairs: 0-0, 1-2, 2-4, . . . Generalize the pattern and express it as y = 2x. Notice that a graph of daily plant growth must leave spaces for unmeasured Saturday and Sunday values. For every 1 km, a constant "up by" $2 in both the y-column of a table and the y-axis in a graph generates a linear pattern (spatial) with a slope of 2 (numeric). Y= 2x can be read from, or produced in, both a table and a graph.
2	• Elaborate initial integrated numeric and spatial understanding to create more sophisticated variations. • Integrate understanding of y = x and y = x + b to form a mental structure for linear functions. • Integrate rational numbers and negative integers. • Form mental structures for other families of functions, such as y = xn + b.	Look at the function below. Could it represent y = x – 10? Why or why not?  If you think it could not, sketch what you think it looks like.

3	• Integrate variant (e.g., linear and nonlinear) structures developed at level 2 to create higher-order structures for understanding more complex functions, such as polynomials and exponential and reciprocal functions. • Elaborate understanding of graphs and negative integers to differentiate the four quadrants of the Cartesian plane. • Understanding the relationship of these quadrants to each other.	At what point would the function $y = 10x - x^2$ cross the x-axis? Please show all of your work.

Appendix D

A Framework for Balance

Source:

Schoenfeld, A. (2007a). Issues and tensions in the assessment of mathematical proficiency. In A. Schoenfeld (Ed.), *Assessing mathematical proficiency*, (pp. 77-97). New York: Cambridge UP.

A Framework for Balance

Mathematical Content Dimension

- *Mathematical content* will include some of:

  Number and quantity including: concepts and representation; computation; estimation and measurement; number theory and general number properties.

  Algebra, patterns and function including: patterns and generalization; functional relationships (including ratio and proportion); graphical and tabular representation; symbolic representation; forming and solving relationships.

  Geometry, shape, and space including: shape, properties of shapes, relationships; spatial representation, visualization and construction; location and movement; transformation and symmetry; trigonometry.

  Handling data, statistics, and probability including: collecting, representing, interpreting data; probability models — experimental and theoretical; simulation.

  Other mathematics including: discrete mathematics, including combinatorics; underpinnings of calculus; mathematical structures.

Mathematical Process Dimension

- *Phases* of problem solving, reasoning and communication will include, as broad categories, some or all of:

  Modeling and formulating;
  Transforming and manipulating;
  Inferring and drawing conclusions;
  Checking and evaluating;
  Reporting.

Task Type Dimensions

- *Task type*: open investigation; nonroutine problem; design; plan; evaluation and recommendation; review and critique; re-presentation of information; technical exercise; definition of concepts.
- *Nonroutineness*: context; mathematical aspects or results; mathematical connections.
- *Openness*: open end with open questions; open middle.
- *Type of goal*: pure mathematics; illustrative application of the mathematics; applied power over the practical situation.
- *Reasoning length:* expected time for the longest section of the task. (An indication of the amount of scaffolding).

Circumstances of Performance Dimensions

- *Task length*: short tasks (5–15 minutes), long tasks (15–60 minutes), extended tasks (several days to several weeks).
- *Modes of presentation*: written; oral; video; computer.
- *Modes of working*: individual; group; mixed.
- *Modes of response*: written; built; spoken; programmed; performed.

Appendix E

Sample Task and Rubric

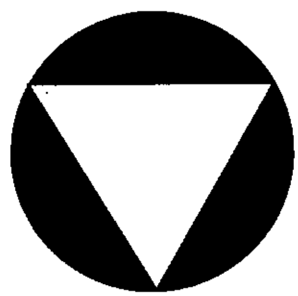
Source:

Burkhardt, H. (2007). Mathematical proficiency: What is important? How can it be measured? In A. Schoenfeld (Ed.), *Assessing mathematical proficiency*, (pp. 77-97). New York: Cambridge UP.

Magazine Cover [Crust 2001–2004] is a *re-presentation of information* task (for grade 3, but adults find it nontrivial). It assesses geometry and mathematical communication.

Magazine Cover

This pattern is to appear on the front cover of the school magazine.



You need to call the magazine editor and describe the pattern as clearly as possible in words so that she can draw it.
Write down what you will say on the phone.

The rubric for Magazine Cover illustrates how complex tasks can, with some scorer training, be *reliably* assessed—as is the practice in most countries and, in the United States, in some of the problems in the Advanced Placement exams.

Magazine Cover: Grade 3	Points
Core element of performance: describe a geometric pattern	
Based on these, credit for specific aspects of performance should be assigned as follows:	
A circle.	1
A triangle.	1
All corners of triangle on (circumference of) circle.	1
Triangle is equilateral. Accept: All sides are equal/the same.	1
Triangle is standing on one corner. Accept: Upside/going down.	1
Describes measurements of circle/triangle.	1
Describes color: black/white.	1
Allow 1 point for each feature up to a maximum of 6 points.	
Total possible points:	6

Appendix F

Guidelines for Formative Feedback

Source:

Shute, V. (2008). Focus on formative feedback. *Review of Educational Research*. *78* (1), 153-189.

TABLE 2

Formative feedback guidelines to enhance learning (things to do)

Prescription	Description and references
Focus feedback on the task, not the learner.	Feedback to the learner should address specific features of his or her work in relation to the task, with suggestions on how to improve (e.g., Butler, 1987; Corbett & Anderson, 2001; Kluger & DeNisi, 1996; Narciss & Huth, 2004).
Provide elaborated feedback to enhance learning.	Feedback should describe the what, how, and why of a given problem. This type of cognitive feedback is typically more effective than verification of results (e.g., Bangert-Drowns et al., 1991; Gilman, 1969; Mason & Bruning, 2001; Narciss & Huth, 2004).
Present elaborated feedback in manageable units.	Provide elaborated feedback in small enough pieces so that it is not overwhelming and discarded (Bransford et al., 2000; Sweller et al., 1998). Presenting too much information may not only result in superficial learning but may also invoke cognitive overload (e.g., Mayer & Moreno, 2002; Phye & Bender, 1989). A stepwise presentation of feedback offers the possibility to control for mistakes and gives learners sufficient information to correct errors on their own.
Be specific and clear with feedback message.	If feedback is not specific or clear, it can impede learning and can frustrate learners (e.g., Moreno, 2004; Williams, 1997). If possible, try to link feedback clearly and specifically to goals and performance (Hoska, 1993; Song & Keller, 2001).
Keep feedback as simple as possible but no simpler (based on learner needs and instructional constraints).	Simple feedback is generally based on one cue (e.g., verification or hint) and complex feedback on multiple cues (e.g., verification, correct response, error analysis). Keep feedback as simple and focused as possible. Generate only enough information to help students and not more. Kulhavy et al. (1985) found that feedback that was too complex did not promote learning compared to simpler feedback.
Reduce uncertainty between performance and goals.	Formative feedback should clarify goals and seek to reduce or remove uncertainty in relation to how well learners are performing on a task, and what needs to be accomplished to attain the goal(s) (e.g., Ashford et al., 2003; Bangert-Drowns et al., 1991).
Give unbiased, objective feedback, written or via computer.	Feedback from a trustworthy source will be considered more seriously than other feedback, which may be disregarded. This may explain why computer-based feedback is often better than human-delivered in some experiments in that perceived biases are eliminated (see Kluger & DeNisi, 1996).
Promote a "learning" goal orientation via feedback.	Formative feedback can be used to alter goal orientation—from a focus on performance to a focus on learning (Hoska, 1993). This can be facilitated by crafting feedback emphasizing that effort yields increased learning and performance, and mistakes are an important part of the learning process (Dweck, 1986).

(continued)

TABLE 2 (*continued*)

Prescription	Description and references
Provide feedback after learners have attempted a solution.	Do not let learners see answers before trying to solve a problem on their own (i.e., presearch availability). Several studies that have controlled presearch availability show a benefit of feedback, whereas studies without such control show inconsistent results (Bangert-Drowns et al., 1991).

TABLE 3

Formative feedback guidelines to enhance learning (things to avoid)

Prescription	Description and references
Do not give normative comparisons.	Feedback should avoid comparisons with other students—directly or indirectly (e.g., "grading on the curve"). In general, do not draw attention to "self" during learning (Kluger & DeNisi, 1996; Wiliam, 2007).
Be cautious about providing overall grades.	Feedback should note areas of strength and provide information on how to improve, as warranted and without overall grading. Wiliam (2007) summarized the following findings: (a) students receiving just grades showed no learning gains, (b) those getting just comments showed large gains, and (c) those with grades and comments showed no gains (likely due to focusing on the grade and ignoring comments). Effective feedback relates to the content of the comments (Butler, 1987; McColskey & Leary, 1985).
Do not present feedback that discourages the learner or threatens the learner's self-esteem.	This prescription is based not only on common sense but also on research reported in Kluger and DeNisi (1996) citing a list of feedback interventions that undermine learning as it draws focus to the "self" and away from the task at hand. In addition, do not provide feedback that is either too controlling or critical of the learner (Baron, 1993; Fedor et al., 2001).
Use "praise" sparingly, if at all.	Kluger & DeNisi (1996), Butler (1987), and others have noted that use of praise as feedback directs the learner's attention to "self," which distracts from the task and consequently from learning.
Try to avoid delivering feedback orally.	This also was addressed in Kluger & DeNisi (1991). When feedback is delivered in a more neutral manner (e.g., written or computer delivered), it is construed as less biased.
Do not interrupt learner with feedback if the learner is actively engaged.	Interrupting a student who is immersed in a task—trying to solve a problem or task on his or her own—can be disruptive to the student and impede learning (Corno & Snow, 1986).

(continued)

TABLE 3 (*continued*)

Prescription	*Description and references*
Avoid using progressive hints that always terminate with the correct answer.	Although hints can be facilitative, they can also be abused, so if they are employed to scaffold learners, provisions to prevent their abuse should be made (e.g., Aleven & Koedinger, 2000; Shute, Woltz, & Regian, 1989). Consider using prompts and cues (i.e., more specific kinds of hints).
Do not limit the mode of feedback presentation to text.	Exploit the potential of multimedia to avoid cognitive overload due to modality effects (e.g., Mayer & Moreno, 2002) and do not default to presenting feedback messages as text. Instead, consider alternative modes of presentation (e.g., acoustic, visual).
Minimize use of extensive error analyses and diagnosis.	In line with findings by Sleeman et al. (1989) and VanLehn et al. (2005), the cost of conducting extensive error analyses and cognitive diagnosis may not provide sufficient benefit to learning. Furthermore, error analyses are rarely complete and not always accurate, thus only helpful in a subset of circumstances.

TABLE 4
Formative feedback guidelines in relation to timing issues

Prescription	Description and references
Design timing of feedback to align with desired outcome.	Feedback can be delivered (or obtained) either immediately or delayed. Immediate feedback can help fix errors in real time, producing greater immediate gains and more efficient learning (Corbett & Anderson, 2001; Mason & Bruning, 2001), but delayed feedback has been associated with better transfer of learning (e.g., Schroth, 1992).
For difficult tasks, use immediate feedback.	When a student is learning a difficult new task (where "difficult" is relative to the learner's capabilities), it is better to use immediate feedback, at least initially (Clariana, 1990). This provides a helpful safety net for the learner so she does not get bogged down and frustrated (Knoblauch & Brannon, 1981).
For relatively simple tasks, use delayed feedback.	When a student is learning a relatively simple task (again, relative to capabilities), it is better to delay feedback to prevent feelings of feedback intrusion and possibly annoyance (Clariana, 1990; Corno, & Snow, 1986).
For retention of procedural or conceptual knowledge, use immediate feedback.	In general, there is wide support for use of immediate feedback to promote learning and performance on verbal, procedural, and even tasks requiring motor skills (Anderson et al., 2001; Azevedo & Bernard, 1995; Corbett & Anderson, 1989, 2001; Dihoff et al., 2003; Phye & Andre, 1989).

(continued)

TABLE 4 (*continued*)

Prescription	Description and references
To promote transfer of learning, consider using delayed feedback.	According to some researchers (e.g., Kulhavy et al., 1985; Schroth, 1992), delayed may be better than immediate feedback for transfer task performance, although initial learning time may be depressed. This needs more research.

TABLE 5
Formative feedback guidelines in relation to learner characteristics

Prescription	Description and references
For high-achieving learners, consider using delayed feedback.	Similar to the Clariana (1990) findings cited in Table 4, high-achieving students may construe a moderate or difficult task as relatively easy and hence benefit by delayed feedback (see also Gaynor, 1981; Roper, 1977).
For low-achieving learners, use immediate feedback.	The argument for low-achieving students is similar to the one above; however, these students need the support of immediate feedback in learning new tasks they may find difficult (see Gaynor, 1981; Mason & Bruning, 2001; Roper, 1977).
For low-achieving learners, use directive (or corrective) feedback.	Novices or struggling students need support and explicit guidance during the learning process (Knoblauch & Brannon, 1981; Moreno, 2004); thus, hints may not be as helpful as more explicit, directive feedback.
For high-achieving learners, use facilitative feedback.	Similar to the above, high-achieving or more motivated students benefit from feedback that challenges them, such as hints, cues, and prompts (Vygotsky, 1987).
For low-achieving learners, use scaffolding.	Provide early support and structure for low-achieving students (or those with low self-efficacy) to improve learning and performance (e.g., Collins et al., 1989; Graesser et al., 2005).
For high-achieving learners, verification feedback may be sufficient.	Hanna (1976) presented findings that suggest that high-achieving students learn more efficiently if permitted to proceed at their own pace. Verification feedback provides the level of information most helpful in this endeavor.

(*continued*)

TABLE 5 (*continued*)

Prescription	Description and references
For low-achieving learners, use correct response and some kind of elaboration feedback.	Using the same rationale as with supplying scaffolding to low-achieving students, the prescription here is to ensure low-achieving students receive a concrete, directive form of feedback support (e.g., Clariana, 1990; Hanna, 1976).
For learners with low learning orientation (or high performance orientation), give specific feedback.	As described in the study by Davis et al. (2005), if students are oriented more toward performance (trying to please others) and less toward learning (trying to achieve an academic goal), provide feedback that is specific and goal directed. Also, keep the learner's eye on the learning goal (Hoska, 1993).